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A theoretical investigation on the magneto-optical spectra of Ce-substituted yttrium aluminium garnet

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Abstract. Calculations of the Faraday rotation and ellipticity spectra at photon energies between 1.6 and 4.0 eV (wavelengths between 775 and 310 nm) resulting from Ce^{3+} ions in Ce-substituted yttrium aluminium garnets (YAG:Ce) based on the quantum theory are presented. The Faraday effect contributed by the Ce^{3+} ions is caused mainly by the intra-ionic electrical dipole transitions between the 4f and 5d configurations. The Faraday effect of the Ce^{3+} ions in both the diamagnetic YAG and the ferrimagnetic yttrium iron garnet (YIG) crystals has a paramagnetic spectral shape, but the resonance frequencies of YAG:Ce are at 2.76 and 3.7 eV, which are obviously higher than those of YIG:Ce. This difference can be explained by the fact that the crystal field acting upon the rare-earth ions in garnets depends strongly on the nature of the next-nearest neighbours. The differences between the transition intensities for the right- and left-handed circularly polarized light from both the lowest and the second crystal-field (CF)-split 5d levels to the ground state are negative. Therefore the Faraday rotation spectrum contributed by the Ce^{3+} ions in YAG has a wave-like shape in the energy region considered. However, for YIG:Ce, the difference between the transition intensities for the right- and left-handed circularly polarized light from the second CF-split 5d level to the ground state is positive. So the corresponding spectrum for YIG:Ce has a different shape. The calculated results are in good agreement with experiment.

1. Introduction

Many experimental investigations on the magneto-optical (MO) and magnetic properties of rare-earth (RE)-substituted iron garnets have been carried out. Recently it has been found that Ce-substituted yttrium iron garnet (YIG:Ce) has a very large Faraday rotation (FR) in the visible and near-infrared regions (Leycuras *et al* 1985, Gomi *et al* 1988, 1990, 1991, Kucera *et al* 1989). The cerium ions give an additional contribution to the FR and Faraday ellipticity spectra of YIG:Ce centred at 1.36 eV (912 nm) and 3.10 eV (400 nm). The intensity of this MO band monotonically increases with increasing cerium concentration in the garnet lattice. The cerium contribution to the FR reaches the value 2×10^4 deg cm^{-1} and -1.3×10^4 deg cm^{-1} at 633 nm and 1150 nm wavelengths, respectively, per single cerium

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ion substituted into a YIG molecule. These values are higher even than those found for Bi^{3+} ions. Hence, particular garnets prepared by RF sputtering, where a high concentration of the Ce^{3+} ions can be achieved (Alex *et al* 1991), may be employed for MO recording.

In order to obtain more insight into the processes responsible for these strong MO effects, comparative studies have been carried out on different magnetic and non-magnetic garnets containing cerium. Kucera (1991a, b) has given detail measurement results on the FR and Faraday ellipticity of Ce-substituted yttrium aluminium garnet (YAG:Ce). It is found that the doping of Ce^{3+} ions into YAG caused pronounced bands to appear in the MO spectra. The structure observed has a paramagnetic spectral shape similar to that of YIG:Ce. However, for YAG:Ce, the resonance frequencies are shifted to a higher energy: 2.76 eV (460 nm) and 3.69 eV (336 nm).

In some of our previous papers (Xu *et al* 1993a, b, Yang *et al* 1994), we have given a theoretical explanation of the strong MO enhancement in YAG:RE based on the quantum theory. It has been shown that the single-ion model is valid for the MO effect caused by the RE sublattice in YIG:RE when the temperature is not very low and the concentration of the RE ions is not very high. The RE contribution to the MO effect is caused mainly by the intra-ionic electric dipole transitions between the $4f^n$ and $4f^{n-1}5d$ configurations of the RE ions which are split by a crystal field (CF) and a superexchange interaction. YIG is a ferrimagnetic compound. In YIG:Ce, there is a superexchange interaction acting upon the Ce^{3+} ions. Therefore, there is a spontaneous FR in YIG:Ce. YAG is a diamagnetic compound. When the concentration of the cerium ions is not very high, there is no superexchange interaction acting on the cerium ions. Therefore, in YAG:Ce, there is no spontaneous FR and the FR emerges only when a magnetic field acts upon it. We think that this difference should not alter the origin of the Faraday effect. So, it is a reasonable assumption that, for YAG:Ce the cerium contribution to the Faraday effect is also caused mainly by the intra-ionic electric dipole transitions between the 4f and 5d configurations of the Ce^{3+} ions.

There is another difference between the Faraday effect in YIG:Ce and that in YAG:Ce, i.e. the resonance frequencies of the FR spectrum for YAG:Ce are shifted towards higher energies. We think that this difference arises because the CF strength in YAG is different from that in YIG. The crystal structure of YAG is the same as that of YIG and the difference between the lattice parameters of these two crystals is not large. Furthermore, the nearest neighbours of the Ce^{3+} ions in YAG are the same as in YIG. Therefore, it might be considered that the CFs upon the cerium ions in these two crystals are nearly the same. However, many experimental results show that the CF upon the RE ions in garnets depends strongly on the nature of the next-nearest neighbours. Some examples follow. Blesse and Bril (1967) have investigated some Ce^{3+}-activated phosphors ($Y_3Al_5O_{12}$:Ce, $Y_3Al_4GaO_{12}$:Ce, $Y_3Al_3Ga_2O_{12}$:Ce, $Y_3Al_2Ga_3O_{12}$:Ce, $Y_{1.5}Gd_{1.5}Al_5O_{12}$:Ce, $YAl_3B_4O_{12}$:Ce, etc) and concluded that the CF splitting is strongly influenced by the nature of the next-nearest neighbours. Grunberg *et al* (1969) obtained the CF parameters for Sm^{3+} and Dy^{3+} ions in YAG and yttrium gallium garnet (YGG). The CFs upon the same ion in these two crystals are very different. The CF upon the Sm^{3+} ions in YAG and YGG obtained by them is also very different from the CF upon the Sm^{3+} ions in YIG obtained by Nekvasil *et al* (1985). From these facts, we think that the CF upon the Ce^{3+} ions in YAG is different from that upon the Ce^{3+} ions in YIG. Thus the difference between the resonance frequencies of the FR spectra for YAG:Ce and YIG:Ce can be explained. From the above considerations, the model and approach used to calculate the MO spectrum for YIG:RE are extended to study the MO behaviour of YAG:Ce.

2. Calculation of Faraday rotation and ellipticity

When the damping can be neglected, the complex FR per unit length caused by the electric dipole transitions is (Crossley *et al* 1969)

$$\varphi = \theta_F + i\psi = \frac{\pi\omega}{c\bar{n}}\frac{(\bar{n}^2+2)^2}{9}(\chi^- - \chi^+). \tag{1}$$

The Lorentz–Lorenz correction is incorporated into this equation. In equation (1), θ_F is the FR, ψ is the Faraday ellipticity, χ^- and χ^+ are the electric susceptibilities for right- and left-handed circularly polarized light, respectively, $\bar{n}$ is the mean refractive index of the material, ω is the angular frequency of the light wave and c is the velocity of the light in vacuum. For YAG, at energies below 2.5 eV, the imaginary part ε_0'' of the diagonal element of the permittivity tensor is approximately zero, the real part ε_0' of the diagonal element is approximately $\bar{n}^2$, and $\bar{n} \simeq 1.828$ (Bond 1965). That is, the index of refraction is very much greater than the extinction coefficient. Furthermore, both ε_0' and ε_0'' are only modestly changed by RE substitution. These features make equation (1) accurate for calculating the MO effect of YAG:Ce in the energy region considered.

By inserting the quantum-mechanical expressions for AC susceptibility into equation (1), we obtain

$$\theta_F = \frac{N\pi(\bar{n}^2+2)^2}{9\bar{n}ch}\sum_{n,g}A_{ng}\frac{\omega^2(\omega_{ng}^2-\omega^2-\Gamma_{ng}^2)}{(\omega_{ng}^2-\omega^2+\Gamma_{ng}^2)^2+4\omega^2\Gamma_{ng}^2}\rho_g \tag{2}$$

$$\psi = \frac{N\pi(\bar{n}^2+2)^2}{9\bar{n}ch}\sum_{n,g}A_{ng}\frac{\omega\Gamma_{ng}(\omega_{ng}^2+\omega^2+\Gamma_{ng}^2)}{(\omega_{ng}^2-\omega^2+\Gamma_{ng}^2)^2+4\omega^2\Gamma_{ng}^2}\rho_g \tag{3}$$

for each type of ion present in the material, where

$$A_{ng} = |\langle n|V_-|g\rangle|^2 - |\langle n|V_+|g\rangle|^2. \tag{4}$$

In equations (2)–(4), N is the number of ions per unit volume, Γ_{ng} is the half-width of resonance lines, $|g\rangle$ is the CF- and applied-magnetic-field-split 4f states with energy E_g, $|n\rangle$ is the CF-split 5d states with energy E_n, $\hbar\omega_{ng} = E_n - E_g$, $V_\pm$ are the electric dipole moment operators for right- and left-handed circularly polarized light, respectively, given by

$$V_\pm = e(x \pm iy) \tag{5}$$

and ρ_g is the occupation probability of the state $|g\rangle$ given by

$$\rho_g = \exp\left(-\frac{E_g}{kT}\right)\Big/\left[\sum_g\exp\left(-\frac{E_g}{kT}\right)\right] = \rho_0\exp\left(-\frac{E_g}{kT}\right). \tag{6}$$

It is necessary to calculate the energy levels split by the CF and magnetic field applied and the corresponding wavefunctions. The perturbation Hamiltonian of a Ce^{3+} ion in the crystal is

$$H' = H_c + \xi(r)\boldsymbol{l}\cdot\boldsymbol{s} + \mu_0\mu_B\boldsymbol{H}\cdot(\boldsymbol{l}+2\boldsymbol{s}) \tag{7}$$

where l is the orbital angular momentum operator, s is the spin angular momentum operator and $\xi(r)$ is the spin–orbit coupling parameter. The third term on the right-hand side is the Zeeman effect Hamiltonian, H is the magnetic field applied, μ_B is a Bohr magneton and H_c is the CF Hamiltonian. The Ce^{3+} ions are at the c sites of the lattice; their environment has D_2 symmetry. So H_c can be expressed as

$$H_c = \sum_k B_{k0} r^k C_0^k(\theta, \varphi) + \sum_{k, q \neq 0} B_{kq} r^k [C_{-q}^k(\theta, \varphi) + C_q^k(\theta, \varphi)] \tag{8}$$

where B_{kq} are CF parameters and $C_q^k(\theta, \varphi)$ have the following relations with the spherical harmonics $Y_q^k(\theta, \varphi)$:

$$C_q^k(\theta, \varphi) = \left(\frac{4\pi}{2k+1}\right)^{1/2} Y_q^k(\theta, \varphi). \tag{9}$$

Because the Zeeman energy induced by the applied magnetic field is very small compared with the spin–orbit interaction and the CF effect, we carried out the perturbation calculation in the following order of priorities: $H_c + \xi(r) l \cdot s$ and $\mu_0 \mu_B H \cdot (l + 2s)$. The CF-split energy levels and the corresponding wavefunctions have been obtained by solving the following secular equation:

$$\|\langle \varphi_i | H_c + \xi(r) l \cdot s | \varphi_j \rangle - E \delta_{ij}\| = 0. \tag{10}$$

In calculating the splitting of the 4f states, both $^2F_{5/2}$ and $^2F_{7/2}$ multiplets have been included. As for the 5d states, the $^2D_{3/2}$ and $^2D_{5/2}$ multiplets have been included.

The choice of the proper set of the parameters of the CF acting upon the Ce^{3+} ions is a crucial step. As mentioned above, the values of the CF parameters depend strongly on the nature of the next-nearest neighbours. So the point-charge model fails to obtain the correct parameters. Grunberg *et al* (1969) determined a set of parameters for the CF acting upon the 4f electrons of the Sm^{3+} ions in YAG by fitting the absorption and emission spectra of the YAG doped with Sm. It is expected that the parameters $B_{kq} \langle r^k \rangle$ of the CF acting upon the RE ions in different YAG:RE compounds change gradually with the atomic number of the RE ions. Therefore, we think that it is reasonable to determine the parameters $B_{kq} \langle r^k \rangle$ of the CF acting upon the 4f electron of the Ce^{3+} ions in YAG:Ce according to the values of CF parameters obtained by Grunberg *et al* for the Sm^{3+} ions in YAG and the ratio of the values of $\langle r^2 \rangle$, $\langle r^4 \rangle$ and $\langle r^6 \rangle$ of the RE ions and then to make a small change to fit the measured values of the FR. The parameters of the CF acting upon the 5d electron of the Ce^{3+} ions are determined by fitting the optical absorption spectrum of YAG:Ce. The non-zero parameters are shown in table 1. The values of ξ for the 4f and 5d configurations and the difference between the mean energies of these two configurations have been taken from the paper by Starostin (1976). The energies of the calculated CF-split levels are shown in table 2.

Table 1, The values of the parameters of the CF acting upon the Ce ions.

	$B_{20}\langle r^2 \rangle$ (cm^{-1})	$B_{22}\langle r^2 \rangle$ (cm^{-1})	$B_{40}\langle r^4 \rangle$ (cm^{-1})	$B_{42}\langle r^4 \rangle$ (cm^{-1})	$B_{44}\langle r^4 \rangle$ (cm^{-1})	$B_{60}\langle r^6 \rangle$ (cm^{-1})	$B_{62}\langle r^6 \rangle$ (cm^{-1})	$B_{64}\langle r^6 \rangle$ (cm^{-1})	$B_{66}\langle r^6 \rangle$ (cm^{-1})
4f	−157.7	273.8	−463.8	2077.6	1245.7	110.9	−403.8	2198	103.7
5d	−1419.2	2254.4	−48 881	74 999	69 412				

Table 2. The values of the CF-split energy levels of the 4f and 5d configurations, taking the average energy of the 4f shell as zero.

	Values of CF-split energy levels					
(cm^{-1})	(cm^{-1})	(cm^{-1})	(cm^{-1})	(cm^{-1})	(cm^{-1})	(cm^{-1})
4f	−1831.84	−1249.71	−1158.58	408.03	842.68	1251.64	1737.78
5d	20 228	28 087	35 641	82 331	82 982		

From table 2, we can see that the Faraday effect comes mainly from the transitions between the lowest CF-split 4f level and the 5d levels. The second and third CF-split 4f levels make small contributions to the Faraday effect. So, in table 3, the wavefunctions of the lowest three CF-split 4f levels are given. In table 4, the calculated 5d levels taking the energy of the lowest CF-split 4f level as zero and the measured 5d levels obtained from the optical absorption spectrum by some workers (Weber 1973, Jacobs *et al* 1978, Miniscalco *et al* 1978) are given. It can be seen that, for the three lowest levels, the calculated values are in good agreement with the measured values. Because the environment of the Ce^{3+} ions in YAG has D_2 symmetry, the 5d states should split into five CF levels. However, only four energy levels have been found in optical measurement. We cannot fit the fourth measured level by changing the CF parameters. We think that the fourth measured level might belong to another excited configuration and the parameters of the CF acting upon the 5d electron determined by us might be reasonable.

Table 3. The energies and the wavefunctions of the lowest three CF-split 4f levels. Here the representation $[J, J_g]$ is used; so, for example, $|\frac{5}{2}, \frac{3}{2}\rangle$ represents the wavefunction $|J = \frac{5}{2}, M_J = \frac{3}{2}\rangle$.

Energy (cm^{-1})	Wavefunction
−1831.84	$-0.51580|\frac{5}{2}, \frac{5}{2}\rangle + 0.59448|\frac{5}{2}, \frac{1}{2}\rangle + 0.60700|\frac{5}{2}, -\frac{3}{2}\rangle + 0.08965|\frac{7}{2}, \frac{5}{2}\rangle$ $+0.05429|\frac{7}{2}, \frac{1}{2}\rangle - 0.00328|\frac{7}{2}, -\frac{3}{2}\rangle - 0.03320|\frac{7}{2}, -\frac{7}{2}\rangle$
−1831.84	$0.60700|\frac{5}{2}, \frac{3}{2}\rangle + 0.59448|\frac{5}{2}, -\frac{1}{2}\rangle - 0.51580|\frac{5}{2}, -\frac{5}{2}\rangle + 0.03320|\frac{7}{2}, \frac{7}{2}\rangle$ $+0.00328|\frac{7}{2}, \frac{3}{2}\rangle - 0.05429|\frac{7}{2}, -\frac{1}{2}\rangle - 0.08965|\frac{7}{2}, -\frac{5}{2}\rangle$
−1249.71	$0.14942|\frac{5}{2}, \frac{5}{2}\rangle - 0.62740|\frac{5}{2}, \frac{1}{2}\rangle + 0.69415|\frac{5}{2}, -\frac{3}{2}\rangle + 0.30258|\frac{7}{2}, \frac{5}{2}\rangle$ $-0.02726|\frac{7}{2}, \frac{1}{2}\rangle - 0.04649|\frac{7}{2}, -\frac{3}{2}\rangle - 0.08794|\frac{7}{2}, -\frac{7}{2}\rangle$
−1249.71	$0.69415|\frac{5}{2}, \frac{3}{2}\rangle - 0.62740|\frac{5}{2}, -\frac{1}{2}\rangle + 0.14942|\frac{5}{2}, -\frac{5}{2}\rangle + 0.08794|\frac{7}{2}, \frac{7}{2}\rangle$ $+0.04649|\frac{7}{2}, \frac{3}{2}\rangle + 0.02767|\frac{7}{2}, -\frac{1}{2}\rangle - 0.30258|\frac{7}{2}, -\frac{5}{2}\rangle$
−1158.58	$0.82518|\frac{5}{2}, \frac{5}{2}\rangle + 0.47204|\frac{5}{2}, \frac{1}{2}\rangle + 0.24875|\frac{5}{2}, -\frac{3}{2}\rangle + 0.00464|\frac{7}{2}, \frac{5}{2}\rangle$ $-0.12224|\frac{7}{2}, \frac{1}{2}\rangle + 0.13783|\frac{7}{2}, -\frac{3}{2}\rangle - 0.02064|\frac{7}{2}, -\frac{7}{2}\rangle$
−1158.58	$0.24875|\frac{5}{2}, \frac{3}{2}\rangle + 0.47204|\frac{5}{2}, -\frac{1}{2}\rangle + 0.82518|\frac{5}{2}, -\frac{5}{2}\rangle + 0.02064|\frac{7}{2}, \frac{7}{2}\rangle$ $-0.13783|\frac{7}{2}, \frac{3}{2}\rangle + 0.12224|\frac{7}{2}, -\frac{1}{2}\rangle - 0.00464|\frac{7}{2}, -\frac{5}{2}\rangle$

The Zeeman splitting of the lowest three CF-split 4f levels caused by the magnetic field applied has been obtained by solving the following secular equation:

$$\|\langle\psi_i|\mu_0\mu_B \boldsymbol{H} \cdot (\boldsymbol{l} + 2\boldsymbol{s})|\psi_j\rangle E\delta_{ij}\| = 0. \tag{11}$$

The external magnetic field used by Kucera (1991a, b) in measuring the Faraday effect in YAG:Ce is 1 T. In order to compare with his measured results, the magnetic field applied is assumed to be 1 T in our calculation. The magnitude of the Zeeman energy is about

Table 4. The calculated values of the CF-split energy levels of the 5d configuration and the measured values of the excited energy levels taking the energy of the lowest CF-split 4f level as zero.

	Values of CF-split energy levels				
	(cm^{-1})	(cm^{-1})	(cm^{-1})	(cm^{-1})	(cm^{-1})
Calculated	22060	29919	37473	84162	84814
	22000	29667	37333	44667	[a]
Measured	22000	30000	37000	44000	[b]
	21765	29647	37059	44706	[c]

[a] Obtained by Weber (1973).
[b] Obtained by Miniscalco *et al* (1978).
[c] Obtained by Jacobs *et al* (1978).

0.5 cm^{-1}, which is very much less than the CF splitting. Therefore the admixing of different CF levels is neglected in solving equation (11).

Because the Ce^{3+} ions are Kramer's ions, every CF-split energy level is doubly degenerate. The external magnetic field splits every CF level into two sublevels. When a plane-polarized electromagnetic wave propagates through the crystal, the transition probabilities for the right- and left-handed circularly polarized light between these sublevels and the 5d levels are the same. Therefore, there will be no Faraday effect if there is no external magnetic field, but the Zeeman splitting makes the occupation probabilities of these sublevels different. This results in a Faraday effect.

The calculated FR spectrum contributed by the Ce^{3+} ions in YAG:Ce contained 0.5 wt.% Ce at 300 K and a magnetic field of 1 T is shown by the solid curve in figure 1. In the calculation, the value of $\langle r \rangle_{4f5d}$ is taken to be 0.561 Å to fit the measured values of the FR. It is a little larger than the corresponding value of the free Ce^{3+} ions given by Starostin (1976), but the corresponding values for free Pr^{3+} and Nd^{3+} ions are 0.476 Å and 0.46 Å, respectively (Judd 1962). So we think that the value used is reasonable. The half-widths Γ_{ng} are determined by fitting the measured Faraday ellipticity and FR spectra. All the optimum values of $\hbar\Gamma$ for the transitions from the lowest three 5d levels to various 4f levels are 0.11 eV. In Kucera's (1991a,b) papers, the values of $\hbar\Gamma$ obtained by the least-squares analysis of the FR and Faraday ellipticity spectra are 0.11 eV and 0.12 eV for the resonance frequencies at 2.76 eV and 3.69 eV, respectively. The values that we used are very close to these values.

As a comparison, the calculated FR spectrum of Y$_2$CeFe$_5$O$_{12}$ at 300 K (Xu *et al* 1993a) is shown by the broken curve in figure 1 as well. Because the magnitude of the FR of Y$_2$CeFe$_5$O$_{12}$ is very different from that of YAG:Ce, the scales of the ordinate for these two samples are different. The scales for YAG:Ce and YIG:Ce are shown on the right- and left-hand sides, respectively, of the figure. It is seen that the FR spectrum of YAG:Ce also has two resonance frequencies below 4 eV. They are at 2.73 eV and 3.7 eV, respectively. They correspond to the transitions from the lowest and the second CF levels of the 5d configuration to the 4f states. For YAG:Ce, besides the total FR, the FRs caused by the transitions from the lowest three 5d CF levels are also shown in figure 1. The FR is contributed mainly by the transitions from the lowest two 5d CF levels to the 4f levels. However, the resonance frequency corresponding to the third 5d CF level is at 4.65 eV; so the contribution of this level to the FR below 4 eV is not very small. Higher 5d CF levels barely contribute to the Faraday effect at the wavelength region considered. It can be seen that, differing from YIG:Ce, the FR spectrum contributed by the Ce^{3+} ions in YAG has a wave-like shape. This difference can be explained as follows. For YIG:Ce, the value of A_{ng} is negative for the
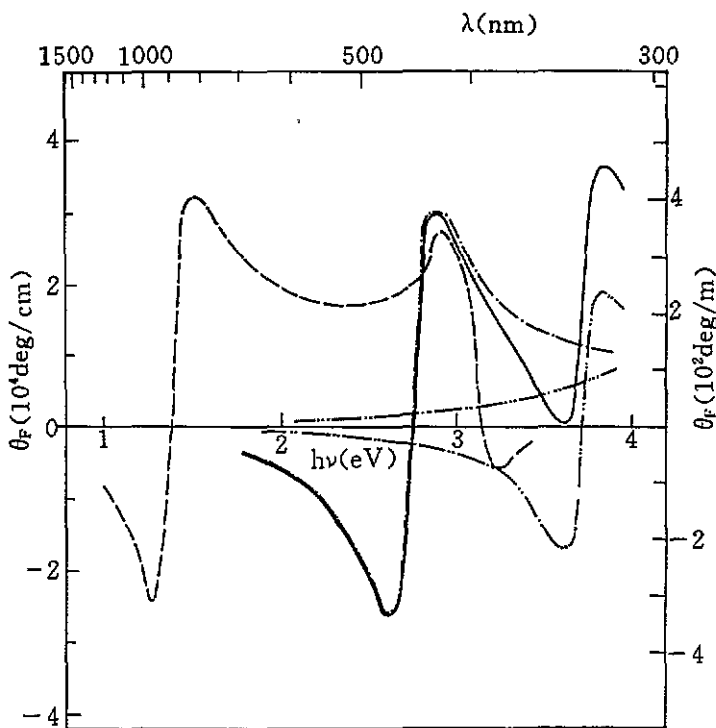
Figure 1. The FR spectrum contributed by the Ce^{3+} ions in YAG:Ce containing 0.5 wt.% Ce at $T = 300$ K and a magnetic field $B = 1$ T and the spontaneous FR spectrum contributed by the Ce^{3+} ions in $Y_2CeFe_5O_{12}$ at 300 K: — · —, FR caused by the lowest 5d level→4f state transitions (the left-hand part of this curve is coincident with the curve of the total FR); — · · —, FR caused by the second 5d level→4f state transitions; — · · · —, FR caused by the third 5d level→4f state transitions; ——, total FR; – – –, FR caused by the Ce^{3+} ions in $Y_2CeFe_5O_{12}$.

transition from the lowest 5d CF level to the ground state (i.e. the lowest CF- and applied-magnetic-field-split 4f state), while it is positive for the transition from the second 5d CF level to the ground state. Therefore the FR is positive for the whole region between these two resonance frequencies and is negative outside this region. However, for YAG:Ce, both the values of A_{ng} for the transitions from the lowest and second 5d CF levels to the ground state are negative. Therefore the FR spectrum has a wave-like shape. The absolute value of A_{ng} for the second 5d CF level is less than that for the lowest 5d CF level. At the same time the corresponding value for the third 5d CF level is positive. Hence the FR has a positive value in the wavelength region from 454.5 nm (2.73 eV) to 335.3 nm (3.7 eV). Valiev *et al* (1989) have measured the Verdet constant of a pure YAG crystal at 300 K in the wavelength region from 280 to 700 nm. From these values we can obtain the FR of pure YAG at 300 K and a magnetic field of 1 T. The theoretical values of the FR in YAG:Ce containing 0.5 wt.% Ce are obtained by summing the corresponding calculated values of the contribution of the Ce^{3+} ions in YAG:Ce and the values for pure YAG. Both of the values of the FR for pure YAG and the theoretical values for YAG:Ce containing 0.5 wt.% Ce are shown in figure 2. The measured values for YAG:Ce in the same conditions as obtained by Kucera (1991a, b) are shown in the figure as well. It can be seen that the calculated values are in good agreement with the measured values in the whole wavelength region considered. The calculated Faraday ellipticity spectrum and the measured spectrum obtained by Kucera (1991a, b) under the

same conditions are shown in figure 3. The calculated ellipticity spectrum is also in good agreement with the measured spectrum.
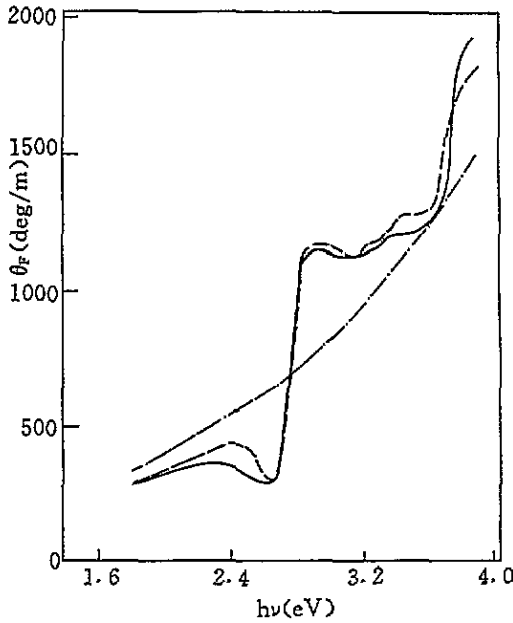

Figure 2. The FR spectrum of YAG:Ce under the same conditions as figure 1: — · —, FR contributed by a pure YAG crystal obtained by Valiev *et al* (1989); ———, theoretical values of the FR in YAG:Ce; – – –, measured values of the FR in YAG:Ce obtained by Kucera (1991a, b) (this curve is coincident with the theoretical curve in the energy region from 2.69 to 2.9 eV).
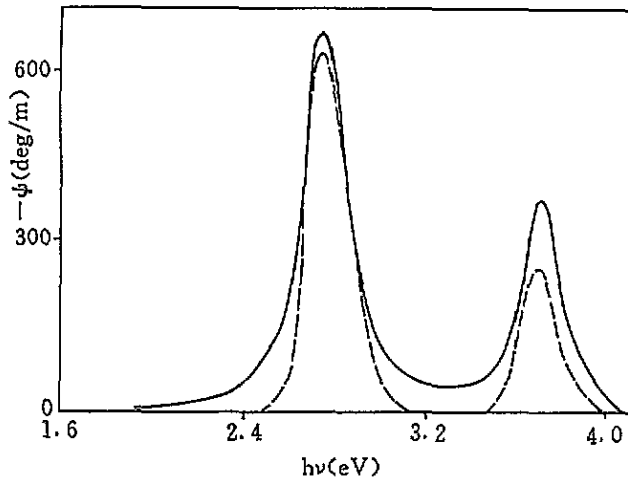

Figure 3. The Faraday ellipticity spectrum of YAG:Ce under the same conditions as figure 1: ———, calculated values; – – –, measured values obtained by Kucera (1991a, b).

3. Conclusion and discussion

From the calculated results, it is seen that the MO behaviour of the Ce^{3+} ions in YAG:Ce is well explained by the model used in this paper. The MO behaviour of the Ce^{3+} ions in YAG is essentially the same as that of the Ce^{3+} in YIG but, because the CF acting upon the RE ions in garnets depends strongly on the nature of the next-nearest neighbours, the resonance frequencies in YAG:Ce are at higher energies. The external magnetic field used in the measurement of the Faraday effect in YAG:Ce is very much less than the molecular field upon the Ce^{3+} ions in YIG. Therefore the observed Faraday effect in YAG:Ce is very much weaker than the spontaneous Faraday effect in YIG:Ce. From our calculated results, we can conclude by a rough estimate that the order of magnitude of the FR in YAG:Ce will be the same as that of the spontaneous FR in YIG:Ce if these two garnets have the same cerium concentration and the magnetic field applied upon YAG:Ce equals the molecular field acting upon the Ce^{3+} ions in YIG. According to the model used in this paper, this result is just as expected.

Acknowledgments

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